Bayesian Analysis for Political Science Workshop

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Day 1

Overview

An Introduction to Me

What to expect from this workshop

By the end of this workshop, you will...

- Understand Bayesian hypothesis testing
- Gain a better understanding of probability and statistics
- Understand the basics of applied Bayesian methods
- Be able to implement Bayesian models and research designs using JAGS

Workshop Schedule

Day 1 Introduction

Day 2 Priors and Posteriors

Day 3 Sampling Methods and Convergence

Day 4 Advanced Bayesian Models

Books and Materials

- Required Texts
  - Gill, Jeff. 2015. *Bayesian Methods: A Social and Behavioral Sciences Approach, Third Edition*. New York: CRC Press.
- Software
  - R
  - JAGS

Today's Agenda

1. Why Bayes?
2. Probability and Bayes' Rule
3. Applying Bayes' Rule
4. Strengths and Weakness of the Bayesian Approach

P-Values

What is the correct interpretation of a p-value?

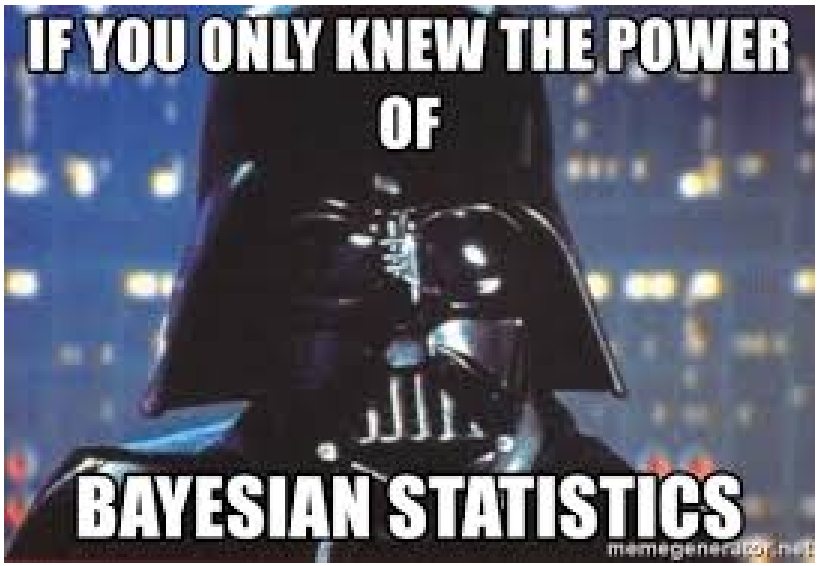
P-Values

What is the correct interpretation of a 95% confidence interval?

Problems with the Frequentist Approach

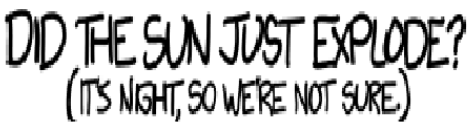
- Relies on asymptotic assumptions
- Lack of replication
- Influence of sample size
- Hidden Assumptions
- Based on a confusing blend of the Likelihood and Frequentist statistical traditions

The Benefits of Bayes

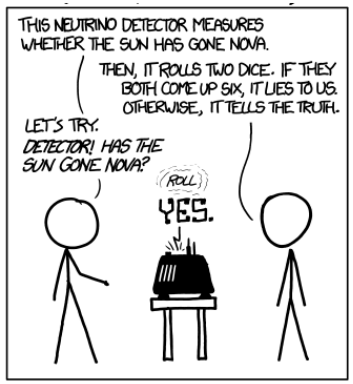


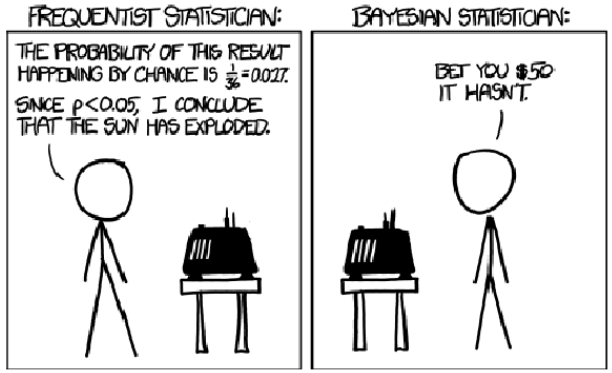
- Overt Assumptions
- Treat probability in the way we naturally think about probability
- Inference for small-N
- Can include prior research directly in the model
- No more dealing with NHST

The Difference Between Bayesians and Frequentists



https://xkcd.com/1132/





Defining Probability

Frequentist Definition

Long run frequency of events.

Bayesian Definition

Expectations of events based on subjective beliefs.

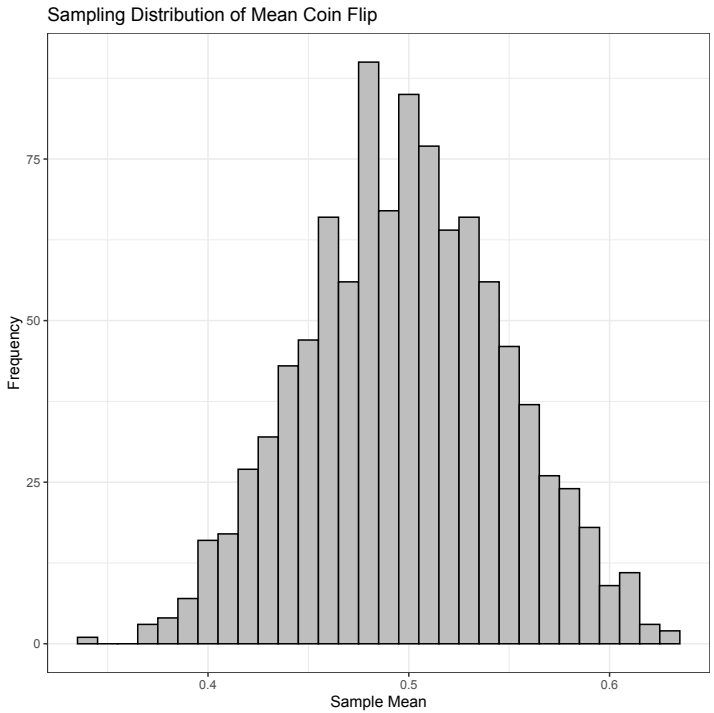
Probability

Think about flipping a coin. What is the probability of it landing on heads?

- One coin flip is a random, unpredictable event
- However, if we flip a coin many times, a pattern emerges that tells us that heads occurs roughly 50% of the time, but not usually exactly 50%.
- But, each set of coin flips can be considered a **sample**.

Probability

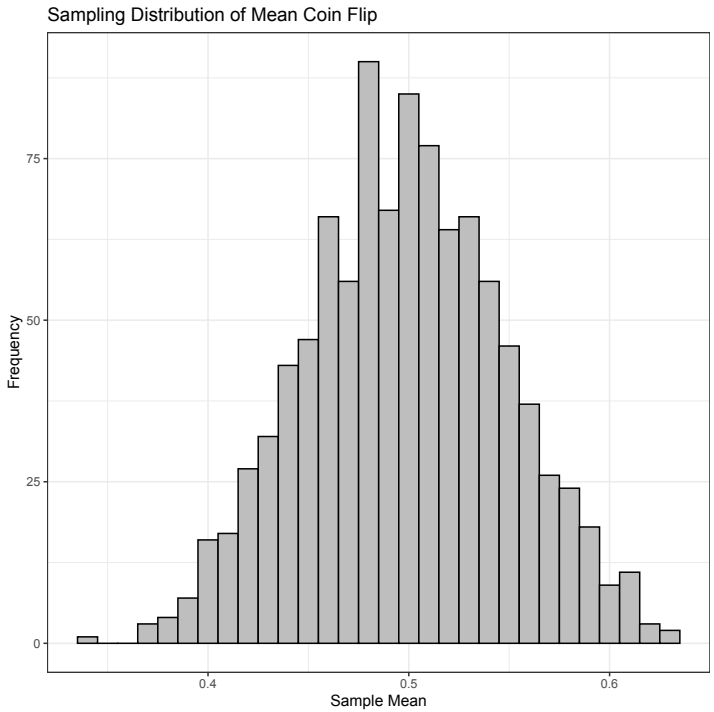
Say we take many samples of 100 coin flips and plot the means of each sample.



- This is a **sampling distribution**
- Not every sample mean is .5, but the mean of the sample means is .49783
- Frequentist statistics relies on this long-run definition of probability

Objective Probability and Hypothesis Testing

If we were testing the hypothesis that we have an unfair coin, we would see how likely our results would be, if they null hypothesis (this is a fair coin) were true.



- The null hypothesis value is .5
- What if our sample mean was .45?
- What if our sample mean was .65?

Objective Probability and Hypothesis Testing

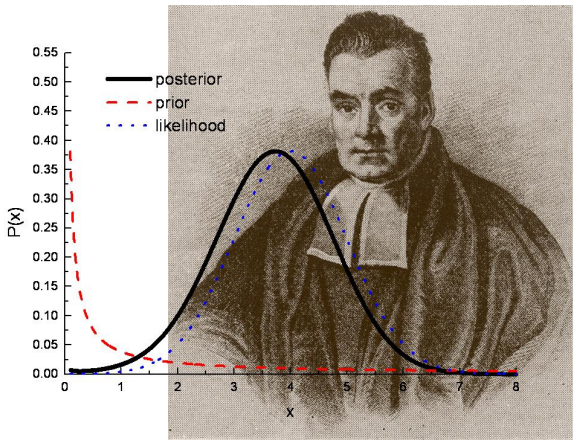
The point of NHST is to see how different our results are from the results expected under the null hypothesis. P-values, then, are the probability we would see our results if the null hypothesis were true.

There is still a chance the null hypothesis is true and you just had a weird sample and you will not know if this is the case.

Jeff Gill

"...frequentist models are rarely presently with caveats such as 'Caution: the scientific conclusions presented here depend on repeated trials that were never performed.' " (23)

Bayesian Probability and Hypothesis Testing



- With Bayes, the **data** are considered fixed and the parameters are considered random.
- The goal is to estimate the **posterior distribution** of the parameters, not a point estimate.
- Bayes uses **prior** information as a baseline and uses the data to update the prior beliefs using the **likelihood**.

Bayesian Probability

Probability Axiom

$$P(A|B) \neq P(B|A)$$

Bayes' Law

$$P(A|B) = \frac{P(B|A) * P(A)}{P(B)}$$

- In 1763, Reverend Thomas Bayes' famous essay solving the conditional probability problem was published posthumously.
- Laplace independently further developed the more detailed statistical foundations of Bayesian statistics (1774, 1781).

Switching the Conditionality in Estimation

Frequentist Statistics Tell us the Probability of the data, given the parameters

$$L(\theta|Y) = \prod_{i=1}^{n} p(Y_i|\theta)$$

Bayes' Law Tells us the Probability of the Parameters, given the data

$$\pi(\theta|X) = \frac{L(\theta|X) * p(\theta)}{\int p(\theta)L(\theta|X)d\theta}$$

Hypothesis Testing with Bayesian Analysis

The Bayes Mantra

$$\pi(\theta|X) \propto L(\theta|X) * p(\theta)$$

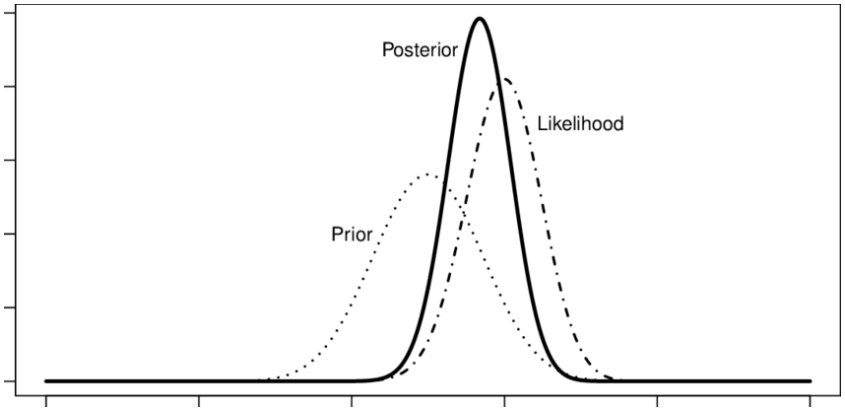


Figure: Source: Etz 2017

Bayesian Statistical Inference

The Frequentist Confidence Interval

- Defined: The interval within which the true parameter can be found in 95% of samples.
- Is about the underlying *population* not about the sample.
- Assumes repeated/ repeatable trials.

The Bayesian Credible Interval

- The interval within which, based on the data, there is a 95% probability the true population parameter is contained.
- This inference is a probability statement about the data, not some underlying population.
- Western and Jackman (1994) argue this is the best way to do inference for comparative data.

So, Why Bayes?

- We don't usually replicate our data.
- Many Comparative and IR scholars use the entire population, rather than samples.
- The data are assumed fixed in Bayes, while the parameters are the random quantities.
- We summarize results with distributions, rather than point estimates.
- We can update our beliefs as we collect new data.
- Bayesian statistics are the basis for machine learning and other advanced statistical models.

Software

Software that does Bayesian Analysis:

- JAGS
- OpenBUGS
- STAN

Software

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- **JAGS**
- BUGS
- STAN

Until Next Time